# 13.2 Channel Coding

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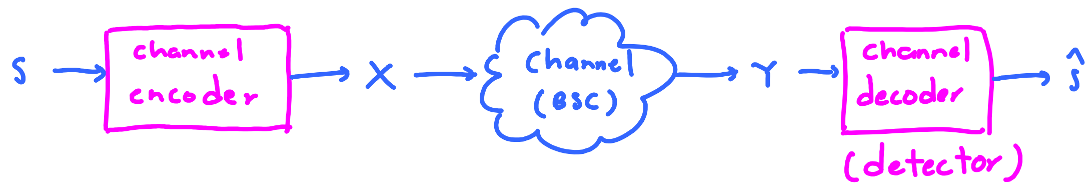
10:34 AM

Review: In 13.1, we studied the $\underbrace{\text{optimal}}_{\substack{\downarrow \\ \text{minimize } \underbrace{\text{probability of error.}}_{P(\mathcal{E})}}}$ detector : MAP detector

## 13.2 Channel Codes




Note: the $S, X, Y, \hat{S}$ may be written as $\underline{S}, \underline{X}, \underline{Y}, \underline{\hat{S}}$ when they are vectors.

There are two classes of channel codes

1) Block codes ← we will focus on this class
2) Convolutional codes ← encoder has memory
(Need: sequential circuit, state diagram, flip-flops, etc. )

**Block codes** : work with $k$ source bits at a time.
(symbols)

$k = 3$ $n = 5$

$\underline{010}\,\underline{101}\,\underline{001}\,\underline{011} \rightarrow \boxed{\text{Ch. Enc.}} \rightarrow 010\underline{0}\underline{1}\,101\underline{1}\underline{0}\,001\underline{0}\underline{1}\,011\underline{1}\underline{0}$

note that the channel encoder maps each block of $k$ source bits into $n$-bit codeword.

code rate $= \frac{k}{n}$ $\qquad$ $n-k = \#$ redundant bits
(used to add redundancy to combat the crossover from BSC.)

Ex. 1 : Repetition code

code rate $= \frac{1}{n}$ $\qquad$ $k=1$

if you repeat each source bit $n$ times

codebook :

| $s$ | $x$ |
|---|---|
| 0 | $\underbrace{00000}_{\text{repeat } n \text{ times}}$ |
| 1 | $\underbrace{11111}_{\text{repeat } n \text{ times}}$ |

For the repetition code above, at the receiver, we may consider several kinds of detectors.

This is what we used in ECS 315.

Detector : ① Majority-vote detector

Ex. receive 01100 $\Rightarrow$ compare

$\# 0\text{'s} > \# 1\text{'s}$

$\Rightarrow \hat{s} = 0$

② MAP detector (HW 7 Q4) ← this is the optimal decoder in terms of $P(\mathcal{E})$

③ Minimum distance detector/decoder
(Hamming)

Let's try this with repetition code

$0 \longrightarrow \underset{\underline{x}^{(0)}}{00000} \longrightarrow$ BSC $\longrightarrow \underset{\underline{y}}{01100}$

$1 \longrightarrow \underset{\underline{x}^{(1)}}{11111} \longrightarrow$ BSC

suppose this is observed.

$\underline{x}$

Hamming distance between two vectors of the same length

$d(\underline{x}, \underline{y})$ $\quad\underline{x}$ $\underline{y}$

is given by ~~#~~ positions at which the elements in $\underline{x}$ are not the same as the elements in $\underline{y}$

$d(00000, 01100) = 2$ ← smaller distance

$d(11111, 01100) = 3$

Minimum distance decoder will calculate

$$\hat{x} = \arg\min_{\underline{x}} d(\underline{x}, \underline{y})$$

and then map back to a value.

So, for $\underline{y} = 01100$, the min distance decoder would say $\hat{x} = 00000$ and $\hat{s} = 0$.

Now that we know minimum distance decoder, we can consider other ways to look at the performance of channel codes (without probability)...

Here, we only care about the codewords $\underline{x}$ and whether we can map the observed channel output $\underline{Y}$ to $\hat{\underline{x}}$ which is the same as $\underline{x}$.
(of course, if $\hat{\underline{x}} = \underline{x}$, then $\hat{\underline{s}} = \underline{s}$.)

There are a couple of quantities that we may consider

1) Error-detecting capability of the code
2) Error-correcting capability of the code

Ex. repetition code

$0 \longrightarrow 00000$ $\quad = 2$ bits

...

$0 \longrightarrow 00000$ $\quad$ $\hookrightarrow = 2$ bits

$1 \longrightarrow 11111$ $\quad$ $\hookrightarrow = 4$ bits

Formal definitions: A channel code is called

a t-error correcting code if it can correct upto t errors from BSC.

a t-error detecting code if it can detect the occurrence of upto t errors from BSC.

Observe that to determine the two quantities above, we use $d_{min}$ which is the minimum Hamming distance among all distinct pairs of codewords in the codebook.

Suppose $d_{min} = d$

the error detecting capability $= d-1$ bits

the error correcting capability $= \left\lfloor \frac{d-1}{2} \right\rfloor$ bits

floor function